Optimization
B.Math. Hons. IInd year
Midsemestral examination
September 2015
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Answer ANY SIX questions.

Q 1. Reduce the matrix below to echelon form by elementary row operations:

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 \\ 3 & 0 & 3 & 0 & 2 \\ 5 & 7 & -9 & 2 & 5 \end{pmatrix}$$

Further, determine its rank.

Q 2. Show that the column space of the matrix $\begin{pmatrix} 2 & 4 & 1 & -1 \\ 3 & 6 & 0 & 1 \\ -1 & -2 & -2 & 3 \end{pmatrix}$ is

$$\{(u, v, v - 2u)^t : u, v \in \mathbf{R}\}.$$

Q 3. Obtain a 2×4 matrix A so that $Ax = 0$ has the general solution

$$\begin{pmatrix} 2\alpha + 3\beta \\ -\alpha \\ 2\beta \\ \alpha - \beta \end{pmatrix}$$

as α, β vary in $\mathbf{R}$.

Q 4. Suppose $A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -2 \\ 2 & 4 & 8 \end{pmatrix}$ and $b \in \mathbf{R}^3$, such that $Ax = b$ is consistent. If $v = (1, -1, 1)^t$, and $u \in \mathbf{R}^3$ is arbitrary, show that $(A + uv^T)x = b$ is consistent.

Q 5. If $A \in M_{m,n}(\mathbf{R})$ is symmetric, show that there exist non-zero real numbers $d_1, \ldots, d_r$ for some $r \leq m, n$, and orthonormal vectors $u_1, u_2, \ldots u_r$ such that

$$A = \sum_{i=1}^{r} d_i u_i u_i^T,$$

$u_i^T u_j = 0$ if $i \neq j$ and 1 otherwise.

Q 6. Consider $\mathbf{C}^3$ with the usual inner product, and let W be the subspace generated by $(1,0,0)^t$ and $(0,1,i)^t$. If $F = W + (1,-1,0)^t$, determine the distance from $(1,1,1)$ to F.
Hint: The projection of a vector on F is the closest point to it on F.

Q 7. Recall that a matrix $A \in M_n(\mathbf{C})$ is said to be normal if $AA^* = A^*A$. Prove that A is unitarily similar to a diagonal matrix.
You may use the fact that every square matrix is unitarily similar to an upper triangular matrix.

Q 8. For the vector space P_2 of polynomials of degree ≤ 2 on $(0,\infty)$, define the inner product

$$< f, g >:= \int_0^\infty xe^{-x} f(x)g(x)dx.$$

Determine an orthonormal basis of P_2.

Q 9. Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Apply the Gram-Schmidt and find an upper triangular matrix B such that BA is orthogonal.